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# MATHEMATICAL MODELS OF INSURANCE AND REINSURANCE

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***Abstract:*** *The topic of the paper is the insurance and reinsurance of enterprises' capital. The notion of capital insurance i.e. capital reinsurance, their classification, as well as the mathematical models used in insurance and reinsurance are discussed here. Reinsurance represents the most efficient form of risk sharing among insurance companies.*

***Keywords:*** *insurance, reinsurance, risk, retention, mathematical models, retention function, compensation function.*

## 1. Introduction

The term insurance refers to security and trust. Therefore, the purpose of insurance is providing security. In broader sense, insurance is joining of all those open to the same danger i.e. risk, together [1]. Risk is an uncertain future event that may cause damaging consequences. Risk is narrowly related to insurance since the main prerequisite for the existence of insurance is the presence of risk. Accordingly, insurance represents the protection of the ownership interests of physical and legal persons during risk realization i.e. insured case, by the insurance funds founded through premium collection from these persons.

In economic terms, insurance represents economic activity that has as its objectives: to estimate the existence of risk that carries material damage to the economic subject (insured) for the realization of an insured event; to cover possible damage caused in the realization of an insured event; to perform the redistribution of damage in time; to realize the recovery of insurance premium in terms of the compensation for damage from the insurance fund.

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Risk represents the uncertainty in terms of the outcome of an accident, the danger against the damaging consequences of which, the economic protection through insurance is organized. In order that the insurance could cover the risk, it must have the following features: the possibility of risk realization, the uncertainty of events to be insured, the danger of the realization of economically damaging event, repetition, that is the risk mustn't be isolated, independence from the will of an insurer and insured, legality of risk, limited scope of risk in space and time and homogeneity.

In practice, it is rare for only one risk event to be insured. It is usually spoken of a risk portfolio. In economic terms, the risk portfolio represents the total ownership of insurance company over insurance policies related to different insured events. In mathematical terms, the risk portfolio represents an ordered $n$-tuples of $n$ elements of different insurance policies.

The insurance company accepts risk from its clients i.e. the insured for compensation called premium. A premium is the amount of money paid by the insured to the insurer in accordance with an insurance contract.

If the risk or risk portfolio is extremely high for an insurance company, it can divide it into smaller parts and pass it over to other companies, its reinsurers. They can further share the risk between their reinsurers. Part of the risk that remains with the insurance company that has taken on the risk from its insured is called retention, [2]. So, the starting risk is covered by an entire net of the insurance and reinsurance of more insurance companies, whereby each has its retention. Determining the size of retention depends on many parameters, that being the issue of actuarial mathematics.

In practice, insurance companies come across different problems, the most common being: evaluation i.e. calculation of a premium; evaluation of risk capital; evaluation i.e. calculation of retention; behavior of the company's management regarding the kinds of risk; maximally acceptable amount of risk for a company; and other. Each problem is very significant for the business of an insurance company. It is in the interest of the company to collect a higher insurance premium in order not to have losses, however, in that case, most of insured would go to another insurance company that has a favorable insurance premium.

The evaluation of risk capital, retention and maximally acceptable amount of risk that a company can cover with its capital are the issues associated with reinsurance. Every insurance company disposes of a certain

amount of capital used for claim payment to its clients. If a company insures an event of high risk whose realization can have catastrophic consequences, the size of the claim for the realization of that event can exceed all the available capital of the company. In that case, the company will share the surplus of the risky capital among its reinsurers and will retain the part that can be covered with available capital. In order to be without the loss, it is in the interest of the insurance company to decide precisely on the part of the risky capital it wants to keep for itself.

## 2. Basic Mathematical Models of Insurance

From the theoretical point of view, a mathematical model can be described as a reservoir. The characteristic feature of this model is that the capital inflow is regular, on one side, while on the other, the capital outflow is unknown. The capital outflow can be very irregular depending on the unpredictable events such as accidents and natural catastrophes. It can be noticed that the stochastic nature of the capital outflow is dual i.e. it is neither known when it will come to a claim, nor what is the size of a claim. This simple model describes the most important aspects of insurance problems.

Mathematical model of insurance depends on four elements, two being stochastic and other two deterministic.The basic elements are:

1) The initial reserves *u* defining the beginning point of the process;
2) The premium return *c* determining capital (profit) accretion;
3) The time sequence when it comes to the claim $T_1$, $T_2$, $T_3$, ..., where $T_1$ refers to the time interval between the beginning moment $t = 0$ and the moment when it comes to the first claim, $T_2$ the time interval between the first and second claim.
4) Sequence of the claim variables $X_1$, $X_2$, $X_3$, ... determining the capital decrease.

It should be pointed out that the sequence $(T_i)$ and $(X_i)$ are the sequences of independent random values.

Apart from the above mentioned values, in insurance theory, the following values are also important :
1) numerical process *N(t)* represents the number of claims up to time *t* and we get

$$N(t) = \sup\{n \geq 1 \mid T_n \leq t\}, \text{ за } t \geq 0,$$

2) the total amount of claims up to time $t$ is defined by the following expression:

$$S(t) = \sum_{i=1}^{N(t)} X_i .$$

When *U (t)* represents the profit of an insurance company we get the following formula:

$$U(t) = u + ct - S(t).$$

Here, the claims $X_1$, $X_2$, $X_3$, ..., and the premium income $c$ are stochastic i.e. random variables.

The greatest importance for the risk have the variables *S* and *N*. When their values are known, the size of the risk is also known. All the other variables can be deduced from these two series of basic variables. This means that the claims distribution $X_i$ should be defined as well as that for time moments $T_i$, when the claims are paid.

The most famous model of modeling risk in insurance theory is the Cramer-Lundberg model. It is a simple model that quite realistically examines the process of the total amount of claims *S*. For the application of this model in practice, it is necessary to meet the following conditions:

1) it comes to the claims at intervals $0 \le T_1 \le T_2 \le \ldots$ that make a homogeneous Puasson process $N(t) = \sup\{n \ge 1 \mid T_n \le t\}$, $t \ge 0$;

2) the claim at time $T_i$ amounts to $X_i$. Sequence $(X_i)$ represents the sequence of independent, identically distributed, non-negative random variables;

3) the sequences $(T_i)$ and $(X_i)$ are independent among themselves, as well as the process *N* and the sequence of random variables $(X_i)$.

The Cramer-Lundberg model, as one of the simplest model of the total amount of claims will be considered in modeling capital reinsurance.For more information on the model se [3], [4], [6].

### 3. Mathematical Model of Reinsurance

Reinsurance represents repeated insurance i.e. insurance of insurance. The insurer can transfer part of the insurance obligations to other